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14. ABSTRACT
The goal of this project was to develop and evaluate methods for nonlinear sensing with collective states of ultracold atoms in optical lattices. Major results include the following: (1) We showed how to use the collapse-and-revival dynamics of interacting atoms to measure m-body interaction strengths with accuracy scaling as n^(m-1/2); m = 1 corresponds to the shot-noise limit. We developed techniques for both m=2 and m=3, the latter exploiting 3-body interactions with super-Heisenberg scaling n^-5/2. (2) We predicted novel spin-dependent 3-body interactions with applications to sensing external magnetic fields. (3) We proposed a method to measure gravitational

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Report Title

Nonlinear Sensing With Collective States of Ultracold Atoms in Optical Lattices

ABSTRACT

The goal of this project was to develop and evaluate methods for nonlinear sensing with collective states of ultracold atoms in optical lattices. Major results include the following: (1) We showed how to use the collapse-and-revival dynamics of interacting atoms to measure m-body interaction strengths with accuracy scaling as n^(m-1/2); m = 1 corresponds to the shot-noise limit. We developed techniques for both m=2 and m=3, the latter exploiting 3-body interactions with super-Heisenberg scaling n^-5/2. (2) We predicted novel spin-dependent 3-body interactions with applications to sensing external magnetic fields. (3) We proposed a method to measure gravitational accelerations (little g) in a very small region of space (e.g., lending itself to atom-chip-based approaches), with potentially long interrogation times. (4) We showed how collapse-and-revival physics can be used to probe Mott insulating and fermionic states. (5) We characterized the effective 3-, 4-, and 5-body interactions between trapped atoms, including universal, effective range, and nonuniversal physics. (6) We developed a dynamical decoupling protocol for removing the influence of 2-body interactions, leaving 3-body interactions dominant. Finally, (7) We characterized Feshbach resonances of magnetic atoms providing results for using spinor atoms to measure magnetic fields accurately.

Enter List of papers submitted or published that acknowledge ARO support from the start of the project to the date of this printing. List the papers, including journal references, in the following categories:

(a) Papers published in peer-reviewed journals (N/A for none)

Received		Paper
03/05/2013	8.00	Philip Johnson, Eite Tiesinga. Quadrature interferometry for nonequilibrium ultracold atoms in optical lattices, Physical Review A, (01 2013): 13423. doi:
03/05/2013	9.00	Indubala I. Satija, Carlos L. Pando, Eite Tiesinga. Soliton dynamics of an atomic spinor condensate on a Ring Lattice, Physical Review A (accepted), (03 2013): 0. doi:
03/10/2015	16.00	X. Y. Yin, Doerte Blume, Philip R. Johnson, Eite Tiesinga. Universal and nonuniversal effective N-body interactions for ultracold harmonically trapped few-atom systems, Physical Review A, (10 2014): 0. doi: 10.1103/PhysRevA.90.043631
03/10/2015	15.00	Khan W. Mahmud, Eite Tiesinga, Philip R. Johnson. Dynamically decoupled three-body interactions with applications to interaction-based quantum metrology, Physical Review A, (10 2014): 0. doi: 10.1103/PhysRevA.90.041602
03/10/2015	17.00	Khan W Mahmud, Lei Jiang, Philip R Johnson, Eite Tiesinga. Collapse and revivals for systems of short-range phase coherence, New Journal of Physics, (10 2014): 0. doi: 10.1088/1367-2630/16/10/103009
03/10/2015	18.00	Khan W. Mahmud, Lei Jiang, Eite Tiesinga, Philip R. Johnson. Bloch oscillations and quench dynamics of interacting bosons in an optical lattice, Physical Review A, (02 2014): 0. doi: 10.1103/PhysRevA.89.023606
04/02/2015	19.00	Alexander Petrov, Eite Tiesinga, Svetlana Kotochigova. Anisotropy-Induced Feshbach Resonances in a Quantum Dipolar Gas of Highly Magnetic Atoms, Physical Review Letters, (09 2012): 0. doi: 10.1103/PhysRevLett.109.103002
06/22/2012	3.00	Chis H. Fleming, Philip R. Johnson, Bei-Lok Hu. Nonequilibrium dynamics of charged particles in a quantized electromagnetic field: causal, stable and self-consistent dynamics from 1/, Journal of Physics A: Mathematical and Theoretical, (06 2012): 255002. doi: 10.1088/1751-8113/45/25/255002
08/08/2012	6.00	Philip R. Johnson, Doerte Blume, Xiangyu Y. Yin, Willam F. Flynn, Eite Tiesinga. Effective renormalized multi-body interactions of harmonically confined ultracold neutral bosons, New Journal of Physics, (05 2012): 0. doi: 10.1088/1367-2630/14/5/053037
08/20/2013	11.00	H. K. Pechkis, J. P. Wrubel, A. Schwettmann, P. F. Griffin, R. Barnett, E. Tiesinga, P. D. Lett. Spinor Dynamics in an Antiferromagnetic Spin-1 Thermal Bose Gas, Physical Review Letters, (07 2013): 0. doi: 10.1103/PhysRevLett.111.025301
08/20/2013	12.00	K. W. Mahmud, E. Tiesinga. Dynamics of spin-1 bosons in an optical lattice: Spin mixing, quantum-phase-revival spectroscopy, and effective three-body interactions, Physical Review A, (08 2013): 0. doi: 10.1103/PhysRevA.88.023602

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Number of Papers published in peer-reviewed journals:

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Received	Paper

TOTAL:

Number of Papers published in non peer-reviewed journals:

(c) Presentations

(1) Kahn Mahmud, "Creating Schrodinger's Cat with Bose-Einstein condensates in double well and multiple wells", Invited Colloquium, George Mason University, Fairfax, VA, September (2012).
(2) E. Tiesinga, "Preparing coherent and squeezed states of ultra-co ld atoms", at George Mason University, Fairfax, VA, Apr. 26, 2012.
(3) E. Tiesinga, "Effective interactions in Ultra-cold few-body systems", Talk at the KITP workshop on Fundamental Science and Applications of Ultra-cold Polar Molecules, January (2013).
(4) P.R. Johnson, "Nonlinear quantum sensing with ultracold atoms in optical lattices", Invited colloquium, Johns Hopkins University (JHU)-Applied Physics Laboratory (APL), February (2013).
(5) Kahn Mahmud, "Dynamics of spin-1 bosons in an optical lattice", Contributed Talk, APS March Meeting, Baltimore, March (2013).
(6) P.R. Johnson, E. Tiesinga, "Quadrature interferometry for nonequilibrium ultracold atoms in optical lattices", Contributed talk, APS March meeting, March (2013).
(7) P.R. Johnson, "Effective multibody interactions of trapped ultracold neutral bosons", NIST Quantum Information and Bose Einstein Condensation (QIBEC) seminar, May (2013).
(8) E. Tiesinga, "Feshbach Resonances and the Control of the Group Velocity of Atoms Propagating Through a Bose Condensate", at the RTG1729-International Conference on Fundamentals and Applications of Ultra-cold Matter, Visselhovede, Germany, September 16, 2013.
(9) E. Tiesinga, "Feshbach Resonances and the Control of the Group Velocity of Atoms Propagating Through a Bose Condensate" at the Institut for Laser physics of the University of Hamburg, September 23, 2013.
(10) P.R. Johnson, "Effective multibody interactions of confined ultracold bosons", Invited Talk, DAMOP, Quebec, June (2013).
(11) P.R. Johnson, E. Tiesinga, "Quadrature interferometry for nonequilibrium ultracold atoms in optical lattices", Contributed Talk, DAMOP, Quebec, June (2013)
(12) E. Tiesinga, Kahn Mahmud "Signature of effective three-body interactions on the dynamics of spin-1 atoms in an optical lattice", Contributed Talk, DAMOP, Quebec, June (2013).
(13) Lei Jiang, E. Tiesinga, "Universal Impurity-Induced Bound State in Topological", Contributed Talk, DAMOP, Quebec, June (2013).
(14) Lei Jiang, Tiesinga, P.R. Johnson, E. Tiesinga, "Collapse and Revival for Double-Well Superlattices", Poster, DAMOP, Quebec, June (2013).
(15) "Controlling the Group Velocity of Colliding Atomic Bose-Einstein Condensates with Feshbach Resonances", at the INT-14-1 workshop ``Universality in Few-Body Systems: Theoretical Challenges and New Directions" held in the Institute for Nuclear Theory, University of Washington, Seattle on April 21, 2014.
(16) E. Tiesinga, "Measurement Science with ultra-cold atoms in optic al lattices," at UCLA Physics and Astronomy, Los Angeles, California on May 6, 2014.
(17) E. Tiesinga, "Measurement Science with ultra-cold atoms in optical lattices," at the Jet Propulsion Laboratory, Pasadena, California on May 9, 2014.
(18) P.R. Johnson "Influence of trap anisotropy and dimensionality on perturbative effective 2- and 3- body interactions", Invited Talk, Institute of Nuclear Theory, Program on Universality in Few Body Physics, University of Washington, Seattle, WA (April, 2014).
(19) E. Tiesinga, "Static and dynamical properties of topological Fulde-Ferrell states in spin-orbit-coupled Fermi gases", at the June 2014, Division of atomic, molecular, and optical physics meeting in Madison, Wisconsin.
(20) E. Tiesinga, "Particle-hole Pair Coherence in Mott insulator quench dynamics" at the June 2014, Division of atomic, molecular, and optical physics meeting in Madison, Wisconsin.
(21) P.R. Johnson, "Influence of trap anisotropy and dimensionality on effective interactions", 2014 Annual Meeting of the Division of Atomic, Molecular, and Optical Physics (DAMOP) of the American Physical Society (APS), Madison, Wisconsin (June 2014).
(22) P.R. Johnson, "Homodyne interferometry for measuring nonequilibrium quantum fields in optical lattices", ICAP 2014, Washington, DC (2014).
(23) E. Tiesinga, "Measurement Science with ultra-cold atoms in an optical lattice" at the conference on "Quantum many-body systems far from equilibrium: Quench dynamics, thermalisation, and cold-atom experiments", March 9, 2015 at the National Institute for Theoretical Physics in Stellenbosch, South Africa.

Number of Presentations: 23.00

Non Peer-Reviewed Conference Proceeding publications (other than abstracts):

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Number of Non Peer-Reviewed Conference Proceeding publications (other than abstracts):

Peer-Reviewed Conference Proceeding publications (other than abstracts):

Received	Paper

TOTAL:

Number of Peer-Reviewed Conference Proceeding publications (other than abstracts):

(d) Manuscripts

Received		Paper
01/23/2014	13.00	Khan W. Mahmud, Lei Jian, Philip R. Johnson, Eite Tiesinga. Particle-Hole Pair Coherence in Mott Insulator Quench Dynamics, P H Y S I C A L R E V I EW L E T T E R S (01 2014)
01/24/2014	14.00	Khan W. Mahmud, Lei Jiang, Eite Tiesinga, Philip R Johnson. Bloch oscillations and quench dynamics of interacting bosons in an optical lattice, PHYSICAL REVIEW A (11 2013)
05/21/2013	10.00	Khan W. Mahmud, Eite Tiesinga. Dynamics of spin-1 bosons in an optical lattice: spin mixing, quantum phase revival spectroscopy and effective three-body interactions , ArXiv e-prints (04 2013)
05/23/2012	1.00	Philip Johnson, Doerte Blume, X.Y. Yin, William Flynn, Eite Tiesinga. Effective renormalized multi-body interactions of harmonically confinedultracold neutral bosons, New Journal of Physics (12 2011)
07/31/2012	5.00	Alexander Petrov, Eite Tiesinga, Svetlana Kotochigova. Anisotropy induced Feshbach resonances in a quantum dipolar gas of highly magneticatoms, Phys Rev Lett (submitted) (03 2012)
09/07/2012	7.00	Eite Tiesinga, Philip Johnson. Quadrature interferometry for nonequilibrium ultracold atoms in optical lattices, Physical Review Letters (submitted) (09 2012)

TOTAL: **6**

Number of Manuscripts:

Books

Received	Book

TOTAL:

Received Book Chapter

TOTAL:

Patents Submitted

Patents Awarded

Awards

Graduate Students

NAME	PERCENT_SUPPORTED
FTE Equivalent:	
Total Number:	

Names of Post Doctorates

NAME	PERCENT_SUPPORTED
Mahmud Khan	0.50
Lei Jiang	0.50
FTE Equivalent:	**1.00**
Total Number:	**2**

Names of Faculty Supported

NAME	PERCENT_SUPPORTED	National Academy Member
Philip Johnson	0.28	
FTE Equivalent:	**0.28**	
Total Number:	**1**	

Names of Under Graduate students supported

NAME	PERCENT_SUPPORTED
FTE Equivalent:	
Total Number:	

Student Metrics

This section only applies to graduating undergraduates supported by this agreement in this reporting period

The number of undergraduates funded by this agreement who graduated during this period: 0.00

The number of undergraduates funded by this agreement who graduated during this period with a degree in science, mathematics, engineering, or technology fields:...... 0.00

The number of undergraduates funded by your agreement who graduated during this period and will continue to pursue a graduate or Ph.D. degree in science, mathematics, engineering, or technology fields:...... 0.00

Number of graduating undergraduates who achieved a 3.5 GPA to 4.0 (4.0 max scale):...... 0.00

Number of graduating undergraduates funded by a DoD funded Center of Excellence grant for Education, Research and Engineering:...... 0.00

The number of undergraduates funded by your agreement who graduated during this period and intend to work for the Department of Defense 0.00

The number of undergraduates funded by your agreement who graduated during this period and will receive scholarships or fellowships for further studies in science, mathematics, engineering or technology fields:...... 0.00

Names of Personnel receiving masters degrees

NAME

Total Number:

Names of personnel receiving PHDs

NAME

Total Number:

Names of other research staff

NAME	PERCENT_SUPPORTED
FTE Equivalent:	
Total Number:	

Sub Contractors (DD882)

Inventions (DD882)

Scientific Progress

Technology Transfer

Final Report for "Nonlinear Sensing With Collective States of Ultracold Atoms in Optical Lattices"

Philip R. Johnson[1] and Eite Tiesinga[2]

[1]*Department of Physics, American University, Washington DC, 20016, USA*
[2]*Joint Quantum Institute, National Institute of Standards and Technology and University of Maryland, Gaithersburg, Maryland 20899, USA*

(Dated: April 2, 2015)

Statement of the problem studied: The primary goal of this project was to develop and evaluate methods for non-linear sensing with collective states of ultracold atoms in optical lattices. We proposed three projects with the purpose of evaluating the promises, challenges, and limitations of sensing in realistic atomic physics systems. An over-arching goal was to find methods for measuring parameters and sensing fields based on the dynamics of interacting collective states and with performance that scales better than standard quantum limit in the number of particles. We mainly focussed on ultracold atoms in optical lattices because these systems offer precise control and confinement of atomic motion, a platform for engineering a wide range of collective states, tunable atom-atom interactions via Feshbach resonances, and the possibility of using effective multibody interactions to exploit higher-body nonlinearities.

Summary of results and publications

Our research under this project appears in eleven publications, which can be organized in three categories: A. Quantum sensing and metrology in optical lattices. B. Characterization and control of atomic interactions. C. Supporting research on other problems. Our list of publications appears in the bibliography.

Category A is quantum sensing and metrology with atoms in optical lattices. We focused on techniques that make use of the coherent superposition states in atom number. These state are not unlike the photon number distribution in a laser, although for atoms the mean atom number $\bar{n}$ in a lattice site is small. Unlike photons, however, atoms interact. Quenching an atomic-superfluid, by suddenly changing from a shallow to deep optical lattice, leads to collapse-and-revival phase dynamics in which the many-body state oscillates between matter-wave and classical (or incoherent) behaviors. We have shown how the collapse-and-revival dynamics of interacting, atom-number superposition states can be used in non-linear measurements, such that the m-body interaction strengths U_m can be measured with an accuracy given by $\delta U_m/U_m \propto 1/(\bar{n})^\gamma$, with inverse power-law coefficient $\gamma > 1$. ($\gamma = 1/2$ corresponds to the usual shot-noise limit.)

A.1. By modifying the usual collapse-and-revival techniques, we showed that for $m = 2$ (two-body interactions) a scaling with $\gamma = 3/2$ can be reached and, moreover, that the order parameter $\langle b \rangle$ (i.e. the expectation value of the quantum field quadrature) rather than its square (i.e. the density operator), can be directly measured [1]. **A.2.** In related work, we showed that spin-dependent coupling strengths, and a novel spin-dependent effective three-body interaction, can be measured via the collapse-and-revival of magnetic spin-1 atoms [2] (highlighted as an "Editors Suggestion").

Our next goal was to identify a method whereby $m > 2$-body interactions could be utilized in a nonlinear quantum sensing technique. Recent analyses had suggested that so-called super-Heisenberg scaling with precision $1/n^m$ (using entangled states) and $1/n^{m-1/2}$ (with coherent states) is possible, but no realizable experimental implementation had been previously described to test these ideas. Note that the ordinary Heisenberg limit $1/n$ is achieved using non-interacting particles ($m = 1$). **A.3.** Our work in [3] (highlighted as an "Editors Suggestion") proposes a method using dynamical decoupling to measure the three-body strength with $1/n^{3/2}$ scaling. This technique is also of interest for its ability to effectively turn off three-body interactions.

Somewhat surprisingly, we have found that collapse-and-revival dynamics are not limited to superfluids and bosonic atoms. **A.4.** In Ref. [4], we show that coherent oscillations can be seen in quenched Mott insulators, and also in metallic fermionic systems. In this work, we describe how the oscillations can be used to detect the presence of short-range coherence in a many-body state. In contrast, the usual collapse-and-revival experiments reveal long-range coherence. Experimental evidence supporting parts of our analysis was published by Will et al. in ["Observation of coherent quench dynamics in a metallic many-body state of fermionic atoms", Nature Communications 6, 6009 (2015)].

Another goal of our research was to study an interferometric method using collective states to measure the acceleration of gravity (little g). **A.5.** In Ref. [5], we describe such a method utilizing Bloch oscillations of an optical lattice superfluid. In this work we estimate that it may be possible to observe up to 50,000 block oscillations in a system, which could enable a new method for high precision measurement of g. An appealing feature of the technique is that the atoms "fall" in momentum space, while they are held in a single region in position space by the optical lattice. This makes possible localized interferometric measurements of g with long interrogation times (in contrast to standard approaches which drop objects over an extended vertical range). Experiments corresponding to aspects of this work were soon after published by Meinert et al. in ["Interaction-induced quantum phase revivals and evidence for the transition to the quantum chaotic regime in 1D atomic Bloch oscillations", PRL 112, 193003 (2014)].

Category B research involved the characterization and control of interactions between ultracold atoms. **B.1.** In collaboration with D. Blume and X.Y. Yin at Washington State University, and published in [6], we calculated the renormalized effective 2-, 3-, and 4-body interactions for N neutral ultracold bosons in the ground state of an isotropic harmonic trap, assuming two-body interactions modeled with the combination of a zero-range and energy-dependent pseudopotential. In this work we discovered that the latter finite-range corrections can be surprisingly important. The work provides foundational results that we use in our efforts to exploit the nonlinearities from effective multibody interactions in lattices.

B.2. In Ref. [7] (highlighted as an "Editors Suggestion"), we extend the methods from [6] to study the role of nonuniversal effective interactions for confined ultracold few-atom systems. In this paper, we explicitly calculated the nonuniversal three-body effective interaction, in addition to the universal four- and five-body effective interactions.

A second method for controlling interactions for nonlinear sensing and metrology is via magnetic Feshbach resonances. **B.3.** In collaboration with A. Petrov and S. Kotochigova of Temple University, in Ref. [8] we characterized the anisotropic nature of Feshbach resonances in the collision between ultracold highly magnetic submerged-shell dysprosium atoms. We

showed that Feshbach resonances in dysprosium collisions can only occur due to couplings to rotating bound states, in contrast to well-studied alkali-metal atom collisions, where the broadest (strongest) Feshbach resonances are hyperfine induced and due to rotation-less bound states. These calculations support efforts in using spinor (i.e. magnetic) atoms as a system to measure weak magnetic fields accurately.

Category C involved research on related problems.**C.1.** First, in Ref. [9], we showed that quantum coherent spin oscillations can even be observed in thermal gases. This result is intriguing, and suggests the possibility of achieving quantum sensing without the stringent requirements of maintaining a cold (quantum degenerate) gas. **C.2**. In Ref. [10], we studied soliton dynamics and the onset to chaotic dynamics of an atomic spin-1 condensate on a ring lattice with a small number of sites. This was a step toward our interest in sensing with mesoscopic magnetic systems. **C.3.** Finally, in Ref. [11], we studied the backreaction of the electromagnetic field on moving atoms, showing how to maintain causality and self-consistent energy balance.

Technical details.

A.1. We developed a feasible, lattice-based nonlinear interferometric scheme for measuring nonlinear coupling parameters with "super-Heisenberg" scaling in atom number [1]. The interferometric technique is similar to homodyne detection in quantum optics, and allows direct measurement of quadrature operators of interacting ultracold atoms held in a periodic, optical-lattice potential. The method starts by preparing a well-characterized superfluid in the optical lattice. Tunneling between lattice sites is then turned off (by quickly increasing the lattice depth) and a spatially-selective π pulse prepares two subsystems, one interacting with atoms in internal state a and one non-interacting (acting as a reference system) with atoms in state b. After a controllable evolution time the two systems are interfered and detected to give the quadrature operators. These operations are illustrated in Figs. (1) and (2).

We showed that for our technique the signal-to-noise of a determination of the atom-atom interaction strength U_2 as a function of mean atom number $\bar{n}$ in a lattice site has "super-Heisenberg" scaling. Recall that the standard quantum limit (or shot-noise limit) scales as $1/\bar{n}^{1/2}$, and the Heisenberg limit scales as $1/\bar{n}$, for measurements involving $\bar{n}$ non-interacting atoms. For our nonlinear interferometer, we find that $\delta U_2/U_2 \propto 1/\bar{n}^{3/2}$, if the initial superfluid is described by a product of coherent atom-number states, one for each lattice site. This gives an explicit example of non-linear parameter estimation in a quantum many-body system and is one of the key results from our program. Figure (3) compares the optimal fractional uncertainty $\delta U_2/U_2$ in the measurement of the atom-atom interaction strength U_2, obtained via quadrature measurement, to the standard quantum limit, as a function of the mean atom-number $\bar{n}$ in each lattice site. The method can be realistically implemented and can be used to experimentally test nonlinear sensing with ultracold atoms.

A.2. Characterization of spin mixing and quantum phase revival spectroscopy of spin-1 or magnetic bosons in an optical lattice. We have predicted the existence of a novel effective spin-dependent three-body interaction that should be easily detectable with revival spectroscopy as well as determined the sensitivity of the spin-1 bosons to external magnetic fields [2] (highlighted as an "Editors Suggestion"). Spinor superfluids and Bose condensates are quantum degenerate atomic gases with an internal spin degree of freedom that combine

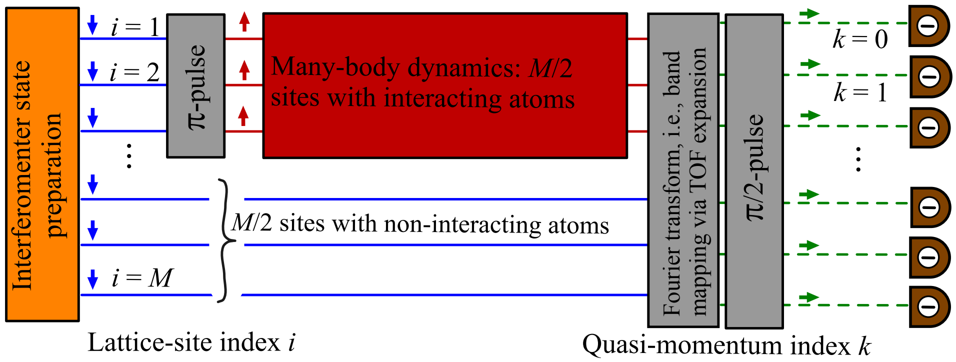

FIG. 1: A schematic of the atomic non-linear interferometer for two-state atoms held in M optical lattice sites. Time evolution is from left to right. The left-most orange box represents the preparation of the initial state. Solid horizontal lines correspond to atoms localized in lattice sites i while dashed lines correspond to atoms in quasi-momenta k after release from the lattice. Red/blue lines are atoms in internal state a/b, while green lines indicate a superposition between the two states. The atomic state is also indicated by small oriented arrows. The gray and red boxes correspond to unitary operations. Detectors (brown) measure the population difference between the two internal states at various quasi-momenta.

magnetism with superfluidity. Examples are optically-trapped spinor condensates of atomic rubidium-87, sodium, or chromium with either spin or angular momentum $f = 1$, 2, or 3 condensates. In contrast to mixtures of two or more atomic states or mixtures of several atomic species, spin-changing collisions in spinor gases permit coherent dynamics among the hyperfine states. In a typical process for two $f = 1$ atoms one in spin component $m = -1$ and one in $m = +1$, can reversibly scatter into two atoms with spin component $m = 0$, which conserves the global magnetization of the condensate. This coherent spin mixing leads, nevertheless, to oscillations of the spin populations, and is an analogue of Josephson oscillations in ultra-cold atoms. Hence, just as collisional interactions allow for a single-component condensate to be spatially coherent, spin-changing collisions, driven by internal interactions, allow coherence among internal degrees of freedom. A positive or negative sign of the strength of the spin-changing interaction determines whether the systems behave anti-ferromagnetic or ferromagnetic, respectively.

We have studied the dynamics of spin-1 atoms in a periodic optical-lattice potential and an external magnetic field in a quantum quench scenario where we start from a superfluid ground state in a shallow lattice potential with small mean-atom number of order one per lattice site and suddenly raise the lattice depth, thereby turning off tunneling between sites. The subsequent time evolution of the non-equilibrium state in each lattice site was simulated exactly and showed collective collapse-and-revival oscillations of matter-wave coherence as well as oscillations in the spin populations. We showed that the complex pattern of these two types of oscillations reveals details about the superfluid and magnetic properties of the initial many-body ground state. The strength of the spin-dependent interaction U_2 can be deduced from the observations. The Hamiltonian that describes the physics of the final deep lattice not only contains two-body interactions but also effective multi-body interactions, which arise due to virtual excitations to higher bands. We derived these effective spin-dependent three-body interaction parameters for spin-1

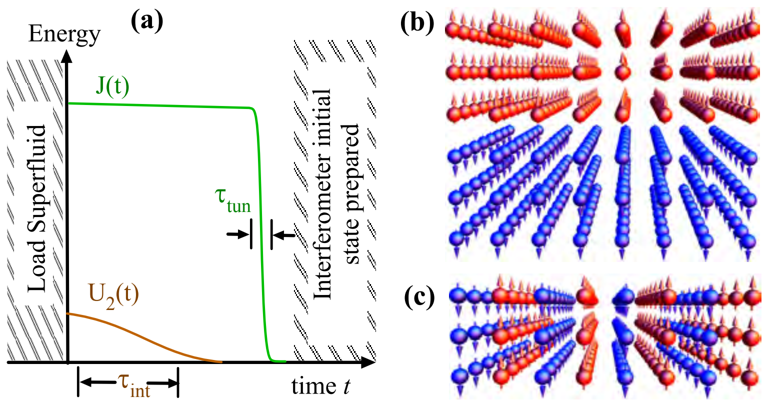

FIG. 2: Implementation of the initial state preparation in Fig. (1). First, a superfluid ground state of atoms in state b is loaded into the optical lattice, with the initial tunneling strength $J(t=0)$ much greater than the two-body interaction strength $U_2(t=0)$. Second, atom-atom interactions are slowly ramped off with timescale $\tau_{\rm int}$ using a Feshbach resonance. Excitation (i.e. from the $k=0$ quasi-momentum) is minimized by choosing $\tau_{\rm int} \gg h/\Delta E_{\rm w}$, where $\Delta E_{\rm w}$ is the level spacing of a weak (harmonic) confining trap that is inevitable in lattice experiments (typically 1-100 Hz). Third, after interactions are ramped-off, tunneling is supressed by quickly increasing the lattice depth with time scale $\tau_{\rm tun}$. Vibrational excitation to higher bands is suppressed by choosing $\tau_{\rm tun} \ll h/\Delta E_{\rm bg}$, where $\Delta E_{\rm bg}$ is the lattice band gap (typically 10-100 kHz). Panels (b) and (c) of Fig. (2) show two possible atom distributions after the site-specific π-pulse (i.e. first beam splitter). In panel (b) atoms in state a (red spheres) and b (blue spheres) are located in the top and bottom half, respectively. This would be probably be the most straightforward implementation. Panel (c) shows another approach, in which identical planes of state a and b atoms alternate; this could be achieved with, for example, a superlattice system.

atoms and described how spin-mixing is affected. Figure 4 shows their dramatic effects. Spinor atoms are unique in the sense that multi-body interactions are directly evident in the *in-situ* number densities in addition to momentum distributions. We have treated both antiferromagnetic (e.g. ^{23}Na atoms) and ferromagnetic (e.g. ^{87}Rb and ^{41}K) condensates.

A.3. Development of a method for changing 2-body versus 3-body interaction strengths, and for realizing super-Heisenberg scaling with $k>2$-body interactions [3]. Both aspects of this work have applications to sensing and metrology with interacting atomic systems. Noting our research on effective multibody interactions which has shown that there are automatically generated effective 3- and higher-body interactions for trapped atom system, the existence of the $k>2$-body interactions poses a challenge for many existing schemes that are formulated under the assumption that there are only 2-body interactions. The simultaneous existence of 2-body interactions also complicates efforts to implement schemes that would exploit $k>2$-body interactions. The method developed in [3] averages out the influence of the 2-body interactions via a dynamical decoupling (or spin-echo) protocol. Exploiting a Feshbach resonance, an applied magnetic field periodically switches the scattering length between positive and negative values, while leaving the 3-body interaction unchanged. This protocol makes it possible to test physics that requires pure (or at least dominant) 3-body interactions. For example, there are predictions for novel

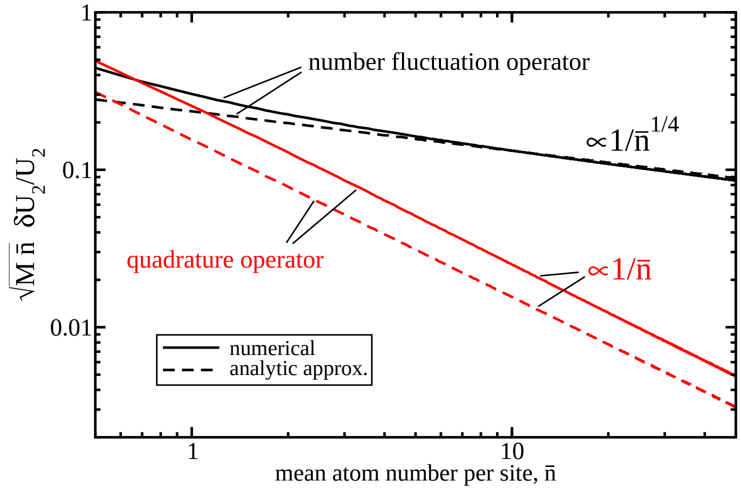

FIG. 3: The optimal fractional uncertainty $\delta U_2/U_2$ in the measurement of the atom-atom interaction strength U_2 as a function of the mean atom-number $\bar{n}$ in each lattice site. The fractional uncertainty has been scaled by the shot-noise limit, $1/\sqrt{M\bar{n}}$, highligthing the improved measurement accuracy even for small mean atom-number $\bar{n}$. Here M is the number of filled lattice sites. We have assumed that initially the wavefunction is a coherent state with mean atom number $\bar{n}$ and that the effective three-body interaction U_3 is zero.

quantum phases for Hubbard models with 3-body interactions. We focused on describing a realizable implementation to test predictions of k-body generated super-Heisenberg scaling precision. We showed via theoretical analysis and numerical modeling that 3-body generated collapse-and-revival dynamics in an optical lattice can yield a $1/n^{3/2}$ scaling in the measurement of the 3-body interaction strength, where n is the number of atoms per lattice site.

A.4. Prediction of the existence of novel collapse-and-revival oscillations that are a distinctive signature of short-range off-diagonal coherence in an optical lattice system [4]. We studied a quench operation that starts with an atomic Mott state in a one-dimensional lattice, and then suddenly raises the lattice depth thereby freezing particle-hole pairs in place, and inducing phase oscillations where the quasi-momentum distribution, revealed through time-of-flight measurements, oscillates between the Γ point (zero quasi-momentum) and the edge of the Brillouin zone. We find a similar effect for fermions and Bose-Fermi mixtures. These results yield a new dynamic probe for strongly correlated states with short-range coherence, complementing previous techniques for detecting long-range coherence. Experimental evidence supporting parts of our analysis was published by Will et al. in [“Observation of coherent quench dynamics in a metallic many-body state of fermionic atoms”, Nature Communications 6, 6009 (2015)].

A.5. The development of a technique to observe gravity-driven Bloch oscillations with lattice-trapped interacting atoms [5]. In addition, we characterized the ability of a superfluid (and thus interacting) gas of atoms to detect accelerations. We have studied the dynamics of spin-less superfluid bosons in a one dimensional vertical optical lattice after a sudden quench

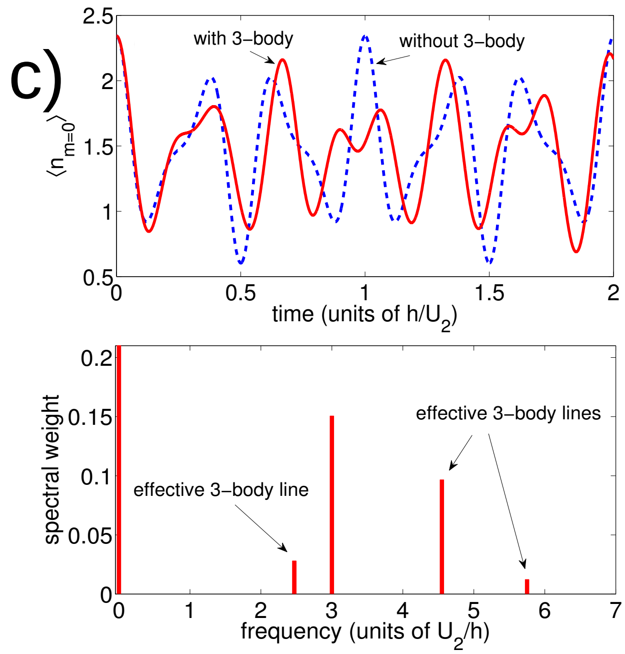

FIG. 4: Effects of effective three-body interactions on the spin-mixing dynamics for a polar (^{23}Na) initial state. (a) The population $\langle \hat{n}_{m=0} \rangle$ in the $m = 0$ state as a function of hold time for occupation $\langle \hat{n} \rangle = 2.35$ and no external magentic field with (solid line) and without (dashed line) the effective three-body interaction, (b) Frequency analysis of the time trace with effective three body interactions reveals the presence of additional frequencies, which could be used to determine the three-body interaction strength.

of the lattice potential, whereby tunneling between sites is turned off. Effectively, the atoms are held in the lattice without moving in position space, but are freely "falling" in quasi-momentum space under the influence of gravity. We have shown that Bloch oscillations in this system can be exploited to measure gravitational acceleration g, and we have also investigated the effects of both weak and strong interactions on these oscillations. Examples of the time evolution of the quasi-momentum distribution $n(k)$ as well as condensate fraction f_c are shown in Fig. 5. The figures on the left and right correspond to strong and weak atom-atom interactions, respectively. Bloch oscillations correspond to the diagonal features with large $n(k)$, most clearly visible in the strong interaction regime and characterized by lines with constant $k - mgt/\hbar$, where g is proportional to the gravitational acceleration. The condensate fraction f_c, which to good approximation follows the maximum value of $n(k)$, shows the effects of the effective three-body interactions as an incommensurate slow oscillation on top of fast oscillations due to the two-body interaction.

An appealing aspect of this system is that because the atoms do not move in real space, both the sensing region and influence of gravitational inhomogeneities can be very small. A significant challenge is maintaining the coherence of the superfluid long enough to allow for precision measurements of gravitational acceleration. We have characterized three important processes that dephase or decohere the superfluid: effective three-body interactions, finite residual tunneling, and residual harmonic trapping. We showed that the center of mass motion due to finite tunneling goes through collapse and revivals as well, thus demonstrating an example of quantum transport where interaction-induced

revivals are important. Finally, we quantified the effects of a residual harmonic trap on the momentum distribution and gave an upper bound for the trapping frequencies below which its effect is negligible.

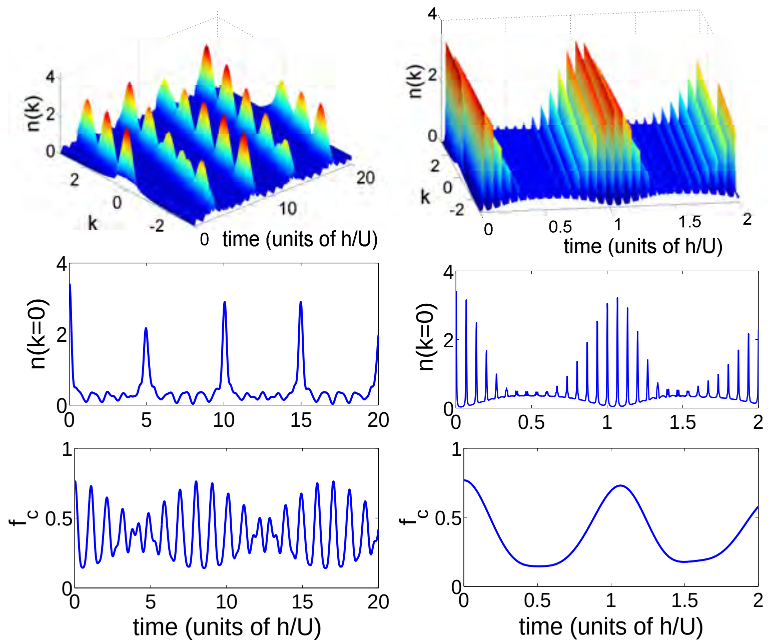

FIG. 5: This simulation includes a three-body interaction with a strength that is one tenth of U_2. The numerical simulations were performed with a time-evolving block decimation algorithm, a method that takes into account quantum correlations.

B.1. In collaboration with D. Blume and X.Y. Yin at Washington State University, and published in [6], we developed a detailed theory of effective multibody interactions for harmonically confined trapped ultracold bosons. We calculated the renormalized effective 2-, 3-, and 4-body interactions for N neutral ultracold bosons in the ground state of an isotropic harmonic trap, assuming two-body interactions modeled with the combination of a zero-range and energy-dependent pseudopotential [2]. We discovered that the latter finite-range corrections can be surprisingly important. Figure (4) shows the 2-, 3-, and 4-body interaction energies as a function of scattering length for typical lattice systems. Figure (5) illustrates one of the processes yielding a contribution to the effective three-body interaction.

Our work provides foundational results that we will use in our efforts to exploit the nonlinearities from effective multibody interactions in lattices. Effective interactions play an important role in interaction-based quantum metrologies. Specifically, we needed to calculate the effective multibody interactions for four reasons: first, because the two-body interactions between lattice-trapped atoms are modified by the confinement and this effect must be taken into account; second, because effective three- and four-body interactions can play both important quantitative and qualitative roles in the dynamics of tightly confined and strongly correlated lattice-trapped bosons; third, to enable our analysis of whether effective k-body interactions with $k > 2$ can be useful for nonlinear interferometry (motivated by previous work in the literature showing that k-body interactions can yield

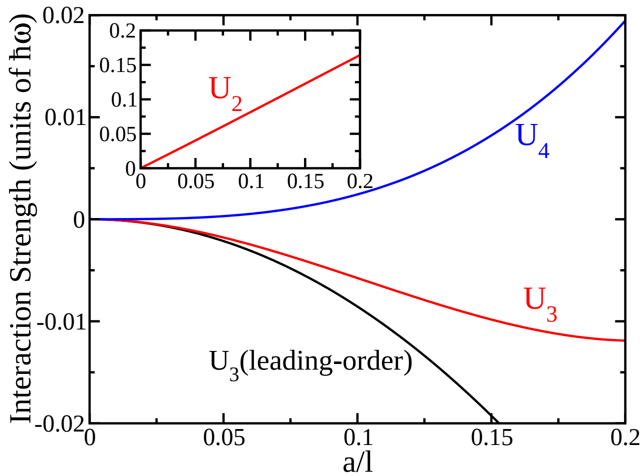

FIG. 6: Predictions for dimensionless effective 2-,3-, and 4-body interaction energies versus a/l, through third order in a/l, where a and l are the scattering and harmonic oscillator lengths, respectively. Energies are in units of harmonic oscillator energies $\hbar\omega$. In Ref. [2], we find $U_3 = -(0.85576...)(a/l)^2 + 2.7921(1)(a/l)^3 + \mathcal{O}(a^4)$ and $U_4 = +(2.43317...)(a/l)^3 + \mathcal{O}(a^4)$ (see Ref. [2] for expressions for U_2, including correction due to finite range nature of the intrinsic 2-body interactions.) The inset shows the first-order behavior of the two-body energy U_2. For experiment with trapped Rb or Cs, a/l is typically $\approx 0.10 - 0.15$. The black line labeled U_3 (leading order) is the second-order result for the 3-body effective interaction; the red and blue lines labeled U_3 and U_4, respectively, show the significant third-order 3- and 4-body corrections for realistic scattering lengths.

optimal scaling in atom number of $1/n^k$); and fourth, to enable our design of effective interactions that couple an atomic systems to external fields (gravitational and magnetic fields).

B.2. In Ref. [7] (highlighted as an “Editors Suggestion”), we extend the methods from [6] to study the role of nonuniversal effective interactions for confined ultracold few-atom systems. In this paper, we explicitly calculated the nonuniversal three-body effective interaction, in addition to the universal four- and five-body effective interactions. The nature of non-universality in few-body systems has been of interest for over 40 years, but only recently have experiments reached the level of being able to probe this physics. On the theoretical side, most effort has been directed at understanding so-called universal properties, such as the Efimov effect. Nonuniversal behaviors, however, are of practical and fundemental importance. On a practical level, they lead to (albeit typically small) corrections that can be important for high-precision metrology and interferometry with atoms. On the fundemental level, detection of nonuniversal properties allows experiments to probe the short-range interactions of atoms. Our work clarifies how and to what extent nonuniversality in three-body systems emerges.

B.3. In collaboration with A. Petrov and S. Kotochigova of Temple University, in Ref. [8], we characterized the anisotropic nature of Feshbach resonances in the collision between ultracold highly magnetic submerged-shell dysprosium atoms. We showed that Feshbach resonances in dysprosium collisions can only occur due to couplings to rotating

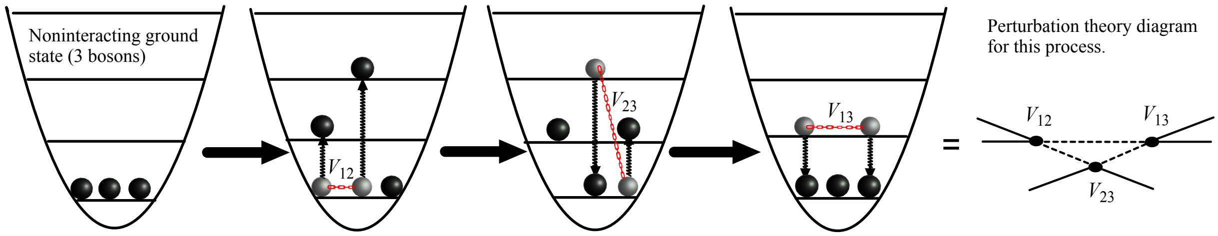

FIG. 7: Sequence of boson-boson interaction induced transitions to higher orbitals. This example generates corrections to the ground state energy that can be viewed as an effective three-body interaction. The process, which involves three interaction vertices, arises at third order in perturbation theory and gives the energy shift $\alpha_3^{(3)}(a/l)^3$, where a is the scattering length, l is the harmonic oscillator length, and $\alpha_3^{(3)} = 0.56494(1)$ (we calculate this coefficient in [2]). Links labelled V_{ij} represent intrinsic 2-body interactions between particles i and j. Black arrows represent virtual transitions to and from excited orbitals. Solid and dashed lines represent atoms in ground and excited vibrational states, respectively. The diagram on the far right shows the perturbation theory diagram for this process.

bound states, in contrast to well-studied alkali-metal atom collisions, where the broadest (strongest) Feshbach resonances are hyperfine induced and due to rotation-less bound states. This is in contrast to well-studied alkali-metal atom collisions, where the broadest (strongest) Feshbach resonances are hyperfine induced and due to rotation-less bound states. Our first-principle coupled-channel calculation of the collisions between these spin-polarized bosonic dysprosium atoms reveals a strong interplay between the anisotropies in the dispersion and magnetic dipole-dipole interaction. The former anisotropy is absent in alkali-metal and chromium collisions. For example, we show in Fig. (6) that both types of anisotropy significantly affect the scattering length or Feshbach spectrum as a function of an external magnetic field. Over a 20 mT magnetic field range we predict about ten Feshbach resonances. These calculations support efforts in using spinor (i.e. magnetic) atoms as a system to measure weak magnetic fields accurately (Project 3 from our proposal). This work also provides another atomic system with a practical method for controlling (and turning off) interactions. These calculations support efforts in using spinor (i.e. magnetic) atoms as a system to measure weak magnetic fields accurately.

C.1. A collaboration with an experimental effort to observe population oscillations in a thermal (non-condensed) gas of spin-1 bosons. This result illustrates that observation of quantum coherent spin-oscillations does not always require quantum degeneracy, and has been published in Physical Review Letters [9]. We helped model experimental observations of coherent spin-population oscillations in a cold thermal, Bose gas of spin-1 ^{23}Na atoms. The population oscillations in a multi-spatial-mode thermal gas have the same behavior as those observed in a single-spatial-mode antiferromagnetic spinor Bose Einstein condensate. We demonstrated this by showing that the two situations are described by the same dynamical equations, with a factor of two change in the spin-dependent interaction coefficient, which results from the change to particles with distinguishable momentum states in the thermal gas. We compared this theory to the measured spin population evolution after times up to a few hundreds of ms, finding quantitative agreement with the amplitude and

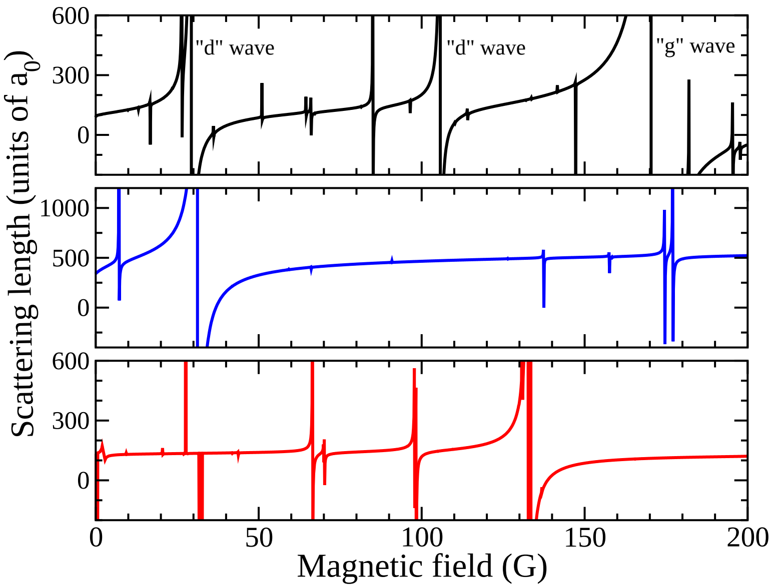

FIG. 8: Scattering length of $m = -8$ ^{164}Dy atoms as a function of magnetic field with and without magnetic dipole-dipole or anisotropic contribution of the dispersion interaction. We assume a collision energy of $E/k = 30$ nK. Channels with even partial waves l up to 10 are included. The top panel shows the case when all interactions are included. At $B = 0$ the scattering length is $89a_0$, where a_0 is the bohr radius. For the three broad resonances the first partial wave for which the resonance appears is shown. The middle and bottom panels are obtained when the dispersion and magnetic dipole-dipole anisotropy is set to zero, respectively.

period. These results are intriguing, and suggest the possibility of achieving quantum sensing without the stringent requirements of maintain a cold (quantum degenerate) gas.

C.2. In Ref. [10], we studied the dynamics of macroscopically-coherent matter waves of an ultra-cold atomic spin-1 or spinor condensate on a ring lattice of six sites and a mesoscopic number of atoms per site, schematically shown in Fig. 9, and demonstrated a novel type of spatio/internal temporal Josephson effect. Using a time-dependent mean-field theory and a discrete solitary mode of uncoupled spin components as an initial condition, the time evolution of this many-body system was found to be characterized by two dominant frequencies leading to quasiperiodic dynamics at various sites. The dynamics of *spatially-averaged* and *spin-averaged* degrees of freedom, however, was periodic enabling an unique identification of the two frequencies. By increasing the spin-dependent atom-atom interaction strength c_2 we observed a resonance state, where the ratio of the two frequencies is an integer and the spin-and-spatial degrees of freedom oscillate in “unison”. (The spin-dependent interaction strength U_2 defined in topic (1) is nontrivially related to c_2. It is roughly given by $U_2 \approx c_2\bar{n}$, where $\bar{n}$ is the mean-atom density in one of the sites along the ring.) Crucially, the resonant state was found to signal the onset to chaotic dynamics characterized by a broad band spectrum. In a ferromagnetic spinor condensate with attractive spin-dependent interactions, the resonance was accompanied by a transition from oscillatory- to rotational-type dynamics as the time evolution of the relative phase of the matter wave of the individual spin projections changes from bounded to unbounded.

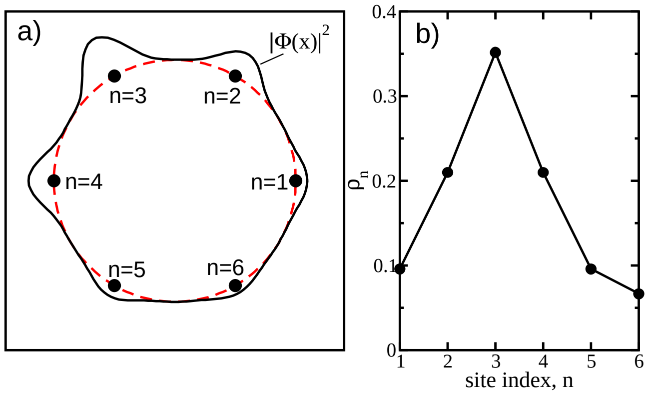

FIG. 9: Panel a) shows a schematic of a ring lattice of six sites (red dashed circle with labeled sites), each containing a $f = 1$ spinor condensate. The solid black line represents the superfluid population along the ring as a polar graph. Panel b) shows the initial fractional population, ρ_n, as a function of site index n used in our dynamical simulations. For each site the initial population of the three magnetic sublevels of the spinor is the same.

C.3. Finally, in collaboration with C.H. Fleming and B.L. Hu of the University of Maryland, in Ref. [11] we studied the backreaction of the electromagnetic field on moving atoms, showing how to maintain causality and self-consistent energy balance. We developed a self-consistent method for obtaining the backreaction (radiation reaction) of the quantized electromagnetic field on charged particles going beyond the dipole and non-relativistic approximation. The approach also consistently includes spin-spin and spin-orbit interactions at the same order of approximation. At this time it appears that the general methods developed are mainly relevant for particles in a quasi-relativistic regime, but it is possible that analogs of the backreaction physics can occur in ultracold systems.

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